- count by 2's

2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 =

2 x 7 = 14

add (repeated addition)

factor

product

Session #2 – February 24th, 2014
Multiplication
Loving God, we pray, that the Toronto Catholic District School Board community continue to witness to the theological virtues of Faith, Hope and Charity.

May we be people of FAITH, always willing to follow the example of Jesus in all that we do.

May we be people of HOPE, always trusting in your loving presence in our lives.

May we be people of CHARITY, always willing to give of ourselves to better the lives of others.

Together, may we be a community of faith, anchored in hope, with heart & charity.

Through, Christ our Lord.
Amen
Who’s Here?

Our Lady of Peace, Our Lady of Wisdom, St Agatha, St Cecilia, St Cyril
Who’s Here?
Our Lady of Peace, Our Lady of Wisdom, St Agatha, St Cecilia, St Cyril
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Our Lady of Peace, Our Lady of Wisdom, St Agatha, St Cecilia, St Cyril

**Our Lady of Peace**
Sonia Correia (gr 2)
Kelly Micco (gr 3)
Anna Maria Mondano (gr 2)
Warnes Keldi (Gr 2)
John Prifti (3)
Gerrard Bonello (VP)

**Our Lady of Wisdom**
Tiffany Rego (gr 2)
Suzan Haggar (gr 2)
Mari-Berje Georgy (gr 3)
Renate Purr (gr 3)
Francesca Polito (gr 3)
(P) Sandra Filicetti
(VP) William Kwon – other school

**St Agatha**
Mireille Fossi-Bakop (gr 2)
Clarisse Tavares M (gr 2)
Nicole Sicotte (3)
Jonathan Remigio (3)

**St. Cecilia**
Kathleen Walsh (gr 2)
Gabrielle Faion-Kralik (gr 2)
Melissa Guitard (gr 3)
Sandra Wilson (gr 3)
Susan Balgavy (gr 2)
Diana Pamula (gr ½)
(P) Lorinda Mazza (D2L day)

**St. Cyril**
Wafaa Ghabour (gr 3)
Mukendi Dibula (gr 3)
Nicole Guegueirre (gr 2)
Marie-Josee Duchesne (gr 2)
(P) Sylvie Talarico

**Superintendents:**
**Math Dept:**
Wilma Simmons, Bart Vanslack

**Ministry:**
What Do WE Want to Study?
SUMMARY of OUR Study Group Learning Goals

● Help students solve multi-step problems even in multiple choice type questions

- Let students problem solve by themselves – make mistakes – analyze it and give them immediate descriptive feedback – allow students to give each other feedback – collaboration

● Build vocabulary – bring back artefacts
What Do WE Want to Study?
Possible question for Inquiry?

● What impact will letting students collaborate, make mistakes and try to solve problems by themselves, have on students’ ability to solve multi-step problems?

● How will analyzing students’ solutions and giving them immediate descriptive feedback, impact their ability to independently solve multi-step problems?

● How will building students’ vocabulary impact their ability to communicate their mathematical thinking in French?
Plan for the Public Research Lesson
- Study and do the math
- Review the main concepts and strategies from research
- Anticipate the success criteria and descriptive feedback based on the lesson learning goal

Co-teach/observe the Public Research Lesson
- Debrief the lesson
- Discuss students’ communication skills – bansho board writing

Reflect on Collaborative Inquiry Question/s
- Reflect on/review our action plan

Collaborate with colleagues
- Share ideas/strategies with grade partners
Preparation for Public Research Lesson
Francesca Gr 3 Class – Multiplication

Les groupes égaux et l’addition répétée

Premiers pas

A. Combien y a-t-il de boîtes de muffins?
   Combien y a-t-il de muffins dans chaque boîte?
   Combien y a-t-il de muffins en tout? Fais une addition.

B. Écris une addition pour chaque sorte d’aliment.

C. Qu’est-ce qui se calcule le plus facilement: le nombre d’épis de maïs ou le nombre de chocolats?
   Explique ta réponse.

D. Choisis un article qui se vend habituellement en groupes égaux de 2, 3, 4 ou 5 unités.
   Dessine le groupe.
   Dessine plusieurs autres groupes semblables.
   Combien d’articles y a-t-il en tout? Écris une addition.

Rappelle-toi!

1. Calcule le nombre total de chaque aliment. Fais une addition.
   a) b)  

2. Écris les 3 prochains nombres de chaque suite.
   a) 5, 10, 15, 20, 25, 30, 35
   b) 10, 20, 30, 40, 50

3. Fais les additions suivantes.
   a) 2 + 2 + 2 + 2
   b) 5 + 5 + 5 + 5 + 5
   c) 4 + 4 + 4 + 4 + 4
   d) 5 + 5 + 5 + 5 + 5

4. a) Compte jusqu’à 10 par bonds de 2.
   b) Quel est le total de 2 + 2 + 2 + 2 + 2?
Les groupes égaux et l'addition répétée

1. muffins
2. cerises
3. œufs
4. épis de maïs
5. boîtes de jus
6. chocolats
7. rouleaux impériaux
8. pains
Francesca’s Gr 3 Class

What’s the mathematical thinking?

A. Pour chaque photo...
- écris le nom de l’aliment: muffin
- quelle est l’addition? \(4 + 4 + 4 + 4 + 4 = 20\)
- il y a _____ groupes de _____.

B. Avec ton(e) partenaire, identifie des articles qu’on trouve en groupes de 2, 3, 4, ou 5. Identifie les articles:
J’ai choisi les Bear paws.

Fais un dessin:

Écris l’addition représenté par ton dessin:
\(30 + 30 = 60\)

A. Pour chaque photo...
- écris le nom de l’aliment: bois de jus
- quelle est l’addition? \(3 + 3 + 3 = 12\)
- il y a _____ groupes de _____.

B. Avec ton(e) partenaire, identifie des articles qu’on trouve en groupes de 2, 3, 4, ou 5. Identifie les articles:
En tout il y a 12 jus d’orange et on a trouvé la réponse par compter par 3 et en tout c’est 12.

Fais un dessin:

Écris l’addition représenté par ton dessin:
\(6 + 3 + 3 = 12\) et maintenant en tout c’est 12.
Francesca’s Gr 3 Class

What’s the mathematical thinking?

1. Écris l’addition représentée par ton dessin:
   - 5 + 5 + 5 = 15
   - Fais un dessin.

2. Écris l’addition représentée par ton dessin:
   - 5 bananes, 3 oranges.
   - Fais un dessin.

3. A. Pour chaque photo, écris le nom de l’aliment:
   - chocolats
   - fruits

4. B. Ainsi, trouve un groupe de 3, 4 ou 5.
   - Identifie les articles:

5. B. Ainsi, trouve un groupe de 3, 4 ou 5.
   - Identifie les articles:

6. A. Pour chaque photo, écris le nom de l’aliment:
   - fruits
   - chocolats

7. A. Pour chaque photo, écris le nom de l’aliment:
   - chocolats
   - fruits

8. A. Pour chaque photo, écris le nom de l’aliment:
   - fruits
   - chocolats

9. A. Pour chaque photo, écris le nom de l’aliment:
   - chocolats
   - fruits

10. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

11. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

12. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

13. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

14. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

15. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

16. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

17. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

18. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

19. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits

20. A. Pour chaque photo, écris le nom de l’aliment:
     - chocolats
     - fruits
Francesca’s Gr 3 Class

What’s the mathematical thinking?
Ms. Polito came to work early today. In the parking lot she saw 6 cars. How many wheels did she see altogether?
What information are we going to use to solve this problem?

Ms. Polito came to work early today. In the parking lot she saw 6 cars. How many wheels did she see altogether?
Break
Let's Do Math As A Teacher

Solve this problem in 2 different ways

1) \(6 \times 4 = 24\)

2) \(\text{Elle a vu 24 roues.}\)

\[
\begin{align*}
4+4 &= 8 \\
4+4 &= 8 \\
8+8+8 &= 24 \\
8+8+8 &= 24 \\
16+8 &= 24 \\
16+8 &= 24
\end{align*}
\]
STUDY - Let’s Do Math As A Teacher
Solve this problem in 2 different ways
MULTIPLICATION
What Ways Do WE Understand Multiplication?

1. What do you notice about Ma and Pa Kettle's mathematical understanding of multiplication?

2. How do you think they learned about multiplication?
What do teachers need to know . . . to teach Multiplication?

- Equal groups
- Repeated addition
- Expectations
- Relationship between multiplication and division
- Strategies
- Counting by 2s, 4s – expose them to multiples
- Relationships between numbers
Operational Sense Trajectory: 1-3

**Grade 1:** – solve a variety of problems involving the addition and subtraction of whole numbers to 20, using concrete materials and drawings (e.g., pictures, number lines) (Sample problem: Miguel has 12 cookies. Seven cookies are chocolate. Use counters to determine how many cookies are not chocolate.);

**Grade 1:** – solve problems involving the addition and subtraction of single-digit whole numbers, using a variety of mental strategies (e.g., one more than, one less than, counting on, counting back, doubles);

**Grade 2** – represent and explain, through investigation using concrete materials and drawings, multiplication as the combining of equal groups (e.g., use counters to show that 3 groups of 2 is equal to 2 + 2 + 2 and to 3 x 2);

**Grade 2** – solve problems involving the addition and subtraction of whole numbers to 18, using a variety of mental math strategies (e.g., “To add 6 + 8, I could double 6 and get 12 and then add 2 more to get 14.”);

* **Grade 2** – count forward by 1’s, 2’s, 5’s, 10’s, and 25’s to 200, using number lines and hundreds charts, starting from multiples of 1, 2, 5, and 10 (e.g., count by 5’s from 15; count by 25’s from 125);

**Grade 3** – relate multiplication of one-digit numbers to real life situations, using a variety of tools and strategies (place objects in equal groups, use arrays, write repeated addition sentences)

**Grade 3** – multiply 7x7 using mental math strategies (ex. Doubles, doubles plus another set, skip counting)

* **Grade 3** – count forwards by 1s, 2s, 5s, 10s, 100s, 25s to 1000
Meanings of Multiplication

At the K to 3 level, students focus only on some meanings of multiplication and division; more meanings are introduced in later grades (combinations, rate/ratio, area, and fractions). The usual meanings for multiplication taught in K to 3 are shown below. Early on, in Kindergarten and Grade 1, the main focus is likely to be on equal groups.

<table>
<thead>
<tr>
<th>MEANING</th>
<th>EXPLANATION AND EXAMPLE</th>
</tr>
</thead>
</table>
| Equal Groups or Sets | $3 \times 4$ is the total number of objects in 3 sets of 4.  
                      | $4 + 4 + 4 = 12$  
                      | $3 \times 4 = 12$  
                      | This model shows both equal groups and repeated addition. |
| Repeated Addition   | The first factor, in this case 3, tells how many times to add the second factor, 4. See the model above.  
                      | $3 \times 4 = 4 + 4 + 4$ |
| An Array            | $3 \times 4$ is the total number of items in a 3-by-4 array.  
                      | $3 \times 4 = 12$  
                      | An array of 3 rows with 4 counters in each row has 12 counters altogether. |

How the Meanings of Multiplication Are Equivalent

It is important for students to understand why the various meanings of multiplication are equivalent so that they can recognize a variety of situations to which multiplication can be applied. One such relationship between the meanings is shown below.

You can determine the number of items in a 3-by-4 array using repeated addition by adding the number in each row, $4 + 4 + 4 = 12$. You can also think of it as 3 equal groups of 4, where the counters in each row make up a group, $3 \times 4 = 12$.  

Relating repeated addition, equal groups, and an array:

3 rows of 4 is 3 groups of 4  
$4 + 4 + 4 = 3 \times 4$
What does Marian say . . . about Multiplication?

Meanings Across the Grades

MULTIPLICATION IN KINDERGARTEN TO GRADE 3

Students are introduced to these meanings:

• repeated addition
  For example,
  \[3 + 3 + 3 + 3 = 4 \times 3 = 12\]

• the total count of a set of equal groups
  For example,
  4 groups of boys with 3 in each is 12 boys:
  \[4 \times 3 = 12\]

• the total count in an array
  For example,
  An array of 4 rows with 3 stamps in each is 12 stamps:
  \[4 \times 3 = 12\]

It is important that students learn to distinguish between an addition situation that can be modelled with multiplication (because all groups are the same size) and one that cannot (where not all groups are the same size).
Marian Small
Multiplication Principles

Principle 1: Multiplication and division “undo” each other. They are related inverse operations.

Multiplication and division are inverse operations; one “undoes” the other. In fact, division is defined as the “opposite” of multiplication. If you start with 12 items and share them among 3 people, each person gets 4 items. How do you get back to where you started (to 12)? You think of the 3 people, each with 4 items, as a multiplication situation, or 3 groups of 4, which is 12.

Principle 2: You can multiply numbers in any order (the commutative property). With division, the order in which you divide the numbers matters.

If you model 3 groups of 4 as shown below, it is not clear why it is the same as 4 groups of 3.

But, if you rearrange the same 12 items in an array, it is obvious why $3 \times 4 = 4 \times 3$. The same model shows both 3 sets of 4 and 4 sets of 3.

- 3 groups of 4 = 12
- 3 rows of 4 = 12
- 4 columns of 3 = 12
- $3 \times 4 = 12$
- $4 \times 3 = 12$

Modelling to explain the commutative property of multiplication.
Math Background: Research and Important Issues

Developing Multiplication Concepts: As with addition and subtraction, the concept of multiplication should be developed first with concrete approaches, gradually becoming more and more abstract. The two most common models for multiplication, the set model and the measurement model, are used first. The set model is represented by objects organized into equal groups (sets), and the measurement model shows multiplication as a series of equal jumps (or measures) on a number line. The array model is also introduced to further develop students' understanding of multiplication. Research tells us that students need extensive experience with these models at the concrete and pictorial stages before they are ready to memorize multiplication facts.

Properties of Whole Number Computations: Although it is not necessary for students at this level to formally name the commutative property of multiplication \((a \times b = b \times a)\) and the distributive property of multiplication \([a \times (b + c) = (a \times b) + (a \times c)]\), students will find these understandings very useful. Both properties are introduced and used frequently throughout the chapter questions.

Recalling Basic Facts: Research indicates that drills are ineffective for teaching recall of basic facts before students have a firm understanding of the procedure. In comparative tests, students who were given frequent oral drills and written tests scored lower than students who learned their basic facts through thinking strategies and games. Lessons 4 and 5 introduce students to two strategies. Several math games are suggested in this Teacher’s Resource in the Extra Support and At Home sections.

Connections to Literature
Add books to your classroom that are related to the math in this chapter. For example:
- *Anna’s Mysterious Multiplying Jar* (Mitsumasa Anno and Masaichiro Anno; Putnam, 1983)
- *What Comes in 2’s, 3’s, and 4’s?* (Suzanne Aker; Simon & Schuster, 1992)
- *The King’s Chessboard* (David Birch; Puffin, 1993)
- *Spaghetti and Marshalls for Ali* (Marilyn Burns; Scholastic, 1997)
- *Two of Everything: A Chinese Folk Tale* (Lily Toy Hong; Whitman, 2003)
- *Sea Squares* (Joy N. Hulme; Little Brown, 1993)
- *Two Ways to Count to Ten* (Ruby Lee; Henry Holt, 1995)
- *Bunches and Bunches of Bunnies* (Louise Matthews; Scholastic Canada, 1989)

Connections to Other Math Strands

**Pattern and Algebra:** Once students complete the multiplication table in Lesson 6, they can find patterns in the rows, columns, and diagonals. Ask students to use a calculator to double a number repeatedly (e.g., 3, 6, 12, 24, 48, 96, 192...), and then discuss the patterns they see in the numbers.

**Measurement:** Students can relate the arrays they have made to the area of each array. Students may begin to notice that the area is equal to the length of the array times the width of the array. Ask students to create as many arrays as possible with 24 objects and to determine the width and the length of each array.

**Data Management and Probability:** In Lesson 2, students read pictographs and determine the value of the scales. Have students gather data from their classmates (e.g., How many books have you read?), and create a pictograph with a many-to-one scale.
STUDYING MATHEMATICS FOR TEACHING

EQAO Analysis

What mathematics in grade 2 and 3 do students need to learn to solve these questions at a level 3 and level 4?

Steven earns $5 for every bundle of newspapers he delivers. He wants to buy a game that costs $18. How many bundles of newspapers does Steven need to deliver to earn enough money to buy this game?

Show your work.

Steven needs to deliver _______ bundles of newspapers.
5 Kyle has 325 trading cards.
He buys 7 more packages of trading cards. Each package contains 5 trading cards.
How many trading cards does Kyle have now?
  ○ 332
  ○ 337
  ○ 355
  ○ 360

6 Marc receives $5 a week for walking a dog.
He wants to buy a video game that costs $42.
How many weeks will it take him to save enough money to buy the video game?
  ○ 5
  ○ 7
  ○ 8
  ○ 9
1. Which number completes the following number sentence?
   \[4 \times 6 = \square\]
   - 10
   - 20
   - 24
   - 28

2. Which number sentence describes the drawing below?
   \[\begin{array}{cccc}
   x & x & x & x \\
   x & x & x & x \\
   x & x & x & x \\
   \end{array}\]
   - \[1 \times 24 = 24\]
   - \[2 \times 12 = 24\]
   - \[4 \times 6 = 24\]
   - \[8 \times 3 = 24\]

3. Which of the following is another way to show \(4 \times 6\)?
   - \[4 + 4 + 4 + 4\]
   - \[6 + 6 + 6 + 6\]
   - \[4 \times 4 \times 4 \times 4\]
   - \[6 \times 6 \times 6 \times 6\]

4. Erik has 24 stamps. What is one way to put his 24 stamps into equal groups?
   - 3 groups of 6
   - 4 groups of 4
   - 5 groups of 6
   - 6 groups of 4
**Math Background**

One meaning for multiplication is *repeated addition* (equal groups that are repeated). One factor tells how many repetitions, and the other factor tells the size of the group or set that is repeated. Therefore, the standard convention in our country is to read $3 \times 5$ as 3 groups of 5 ($5 + 5 + 5$). (In other countries, the convention is to read it as $3 + 3 + 3 + 3 + 3$.)

---

**Lyn’s Multiplication**

Lyn repairs bicycles. It takes her 1 minute to fill each tire on a bicycle.

**Question:** Can Lyn fill the tires on 7 bicycles in 15 minutes?

---

**Lyn’s Multiplication Sentence:**

There are 7 groups of 2 tires. I can model the tires.

$$2 + 2 + 2 + 2 + 2 + 2 + 2 = 14$$

7 groups of $2 = 14$

I can skip count by 2s on a number line.

---

**Multiplication Facts:**

- **Multiplication:** Repeated addition
- **Factors:** Numbers you multiply
- **Product:** The result when you multiply

---

**Sentence:**

It says 7 times 2 equals 14.

**Sentence:**

I can fill all the tires in less than 15 minutes.
Ms. Polito came to work early today. In the parking lot she saw 6 cars. How many wheels did she see altogether?

Questions
– should we give them the pictures or not?
-- should we have only one type of vehicle?
Grade 3

What’s our Lesson Learning Goal?

Expectation:
- relate multiplication of one-digit numbers to real life situations, using a variety of tools and strategies (place objects in equal groups, use arrays, write repeated addition sentences)
- multiply 7x7 using mental math strategies (eg. Doubles, doubles plus another set, skip counting)

Lesson Learning Goal/Success Criteria: Let’s unpack the expectation
What’s the relationship between the curriculum expectations, lesson learning goal and success criteria?

What are the important concepts and strategies for this lesson?
Math Background

One meaning for multiplication is repeated addition (equal groups that are repeated). One factor tells how many repetitions, and the other factor tells the size of the group or set that is repeated. Therefore, the standard convention in our country is to read $3 \times 5$ as 3 groups of 5 ($5 + 5 + 5$). (In other countries, the convention is to read it as $3 + 3 + 3 + 3 + 3$.)

Closing (Whole Class)

Have students summarize their learning by asking them to record in their journals the story problem they wrote for Question 7. Ask them to also represent their story using the four ways of representation used in this lesson (a picture, a number line, an addition sentence, and a multiplication sentence).

Reflecting

1. How are Lyn’s multiplication sentence and addition sentence alike? How are they different?

2. a) Model 1 group of 7.
   b) Show 1 group of 7 on a number line.
   c) Write the multiplication sentence. Describe each part.
   d) Explain why it’s easy to multiply by 1.

Assessment for Feedback

<table>
<thead>
<tr>
<th>Students will</th>
<th>What You Will See Students Doing...</th>
</tr>
</thead>
<tbody>
<tr>
<td>use objects and drawings to model multiplication facts</td>
<td>Students will model multiplication facts correctly using concrete materials, drawings, and number lines.</td>
</tr>
<tr>
<td>use number sentences to represent multiplication concepts</td>
<td>Students can represent a multiplication concept using repeated addition and a multiplication sentence.</td>
</tr>
</tbody>
</table>

If Students Misunderstand

- Students may have less difficulty with concrete materials than with other types of models. Have them move between the different types of models in various ways. (For example, they can model an equation using counters and write an equation from a number line.)
- If students have difficulty translating multiplication concepts represented by a model, they should first write what they see in words (4 groups of 3) and then write the symbols directly underneath.
Grade 3

What’s our Lesson Learning Goal?

Expectation:

• relate multiplication of one-digit numbers to real life situations, using a variety of tools and strategies (place objects in equal groups, use arrays, write repeated addition sentences)
• multiply 7x7 using mental math strategies (eg. Doubles, doubles plus another set, skip counting)

Lesson Learning Goal/Success Criteria: Let’s unpack the expectation

We multiply by: \((6\times4 = 24)\)
- Placing objects in equal groups – 6 cars (equal groups) with 4 wheels each
- Adding the same number \(4+4+4+4+4 + 4\) ( # of wheels ) as many times as there are groups (6 cars)
- Skip counting by 4s - 4, 8, 12, 16, 20, 24
This is what we decided a group . . .

Success Criteria:

We multiply by:

- Making equal groups of objects
- Adding the same number of objects as many times as the number of groups
Ontario 3-Part Problem Solving
Lesson Structure

1- Before (Getting Started) - 5 to 10 minutes – revisiting mathematical ideas and strategies from a previous lesson that relates to the learning goal of the lesson.

2- During (Working On It) - 15 to 20 minutes - solving the lesson problem in pairs, small groups, or individually.

3a- After (Consolidation) - 20 to 25 minutes -> coordination of whole class discussion/analysis of student solutions; co-construction of success criteria.

3b- After (Highlights/Summary) - 5 minutes -> recounting key mathematical ideas and strategies related to the learning goal of the lesson (summary of co-constructed success criteria).

3c -After (Practice) - 5 to 10 minutes – solving a problem that is similar to the lesson problem in order to practise applying new ideas and strategies.
### Specific Curriculum Expectations

**Grade 3**
- relate multiplication of one-digit numbers to real life situations, using a variety of tools and strategies (place objects in equal groups, use arrays, write repeated addition sentences)
- multiply $7 \times 7$ using mental math strategies (ex. Doubles, doubles plus another set, skip counting)

### Lesson Learning Goal (Grade 3)
We multiply by:
- Skip counting to count objects in equal groups
- Count equal groups with repeated addition
- We skip count or count on a number line

### Before (Getting Started)
(task/problem, 2 solutions, annotations)
- Draw equal groups on the board and everything they say. Ask which one is the easier to count?

### During (Working on It)
(problem and problem criteria)
- Ms. Polito came to work early today. In the parking lot she saw 6 cars. How many wheels did she see altogether?

### After (Consolidation)
(4 solutions (labelled, 1, 2, 3, 4) and math annotations)

### After (Highlights/Summary)
(anticipated co-constructed success criteria)
We multiply by: $(6 \times 4 = 24)$
- Placing objects in equal groups
  - 6 cars (equal groups) with 4 wheels each
    (Draw 6 groups with 4 in each)
- Adding the same number
  - 4 - 4 - 4 - 4 - 4 - 4 (number of wheels)
    as many times as there are groups (6 cars)
- Skip counting by 4s - 4, 8, 12, 16, 20, 24

### After (Practice)
(problem and 2 solutions)

#### Chapter 9 Lesson 1

Practising
1. Draw 2 ways to find the total. Write an addition sentence and a multiplication sentence for each:
   - 6 groups of 2 hand grip
   - 4 groups of 4 brake pads

2. We have 6 cars. Each car has 4 wheels. All the cars have 24 wheels.
BEFORE (Getting Started)

- 5 to 10 minutes only
- Activating students’ mathematical knowledge and experience that directly relates to the mathematics in the lesson problem
- Includes student responses to a prompt/problem that is similar to previous work to highlight a few key ideas and/or strategies

**Grade 3 Public Research Lesson**

Why is this task useful?

- Record everything they say on the board. Our goal is to represent equal groups.

KKZ, 2011
Ms. Polito came to work early today. In the parking lot she saw 6 cars. How many wheels did she see altogether?
Understanding Students’ Mathematical Thinking – Work Analysis

1. What mathematics (i.e., concept, algorithm, strategy, model of representation) are the students using in their solution?
   - How does this mathematics in the solution relate to the mathematics lesson learning goal?

2. Which solutions are conceptually-based?
   - Which solutions have an efficient method or algorithm?
     Which solutions include or have the potential for a mathematical generalization?

3. How are the solutions related to each other, mathematically?
AFTER (Highlights/Summary)

• 5 minutes
• Summary of co-constructed success criteria
• Teacher revisits in the different solutions the key ideas, strategies, and models of representation that are related to the lesson learning goal
• Teacher records key ideas, strategies, and models of representation separately, so the students see the explicit focus of learning from the lesson

Grade 2/3 Public Research Lesson

We multiply by:

- Making equal groups of objects
- Adding the same number of objects as many times as the number of groups
AFTER (Practice)

- 5 to 10 minutes
- Teachers chooses 2 or 3 problems, similar to the lesson problem for students to solve individually (or in pairs as a scaffold).
- Problems are different by number (choice, size), problem contexts, or variation of unknown that needs to be solved
- Students are asked to use a strategy different from the one they used in the lesson to solve the practice problems

Grade 3 Public Research Lesson

Practising

4. Show 2 ways to find the total. Write an addition sentence and a multiplication sentence for each.
   - a) 6 groups of 2 hand grips
   - b) 5 groups of 4 brake pads

5. Write each multiplication sentence.
   - a) 6 groups of 2 hand grips
   - b) 5 groups of 4 brake pads

Nelson Chapter 9 Lesson 1
Today's Co-Teaching Public Research Lesson
Co-Teachers, Insider and Outsider Researchers

Co-Teaching and Public Research Teams:

- **Lead Voice** – Francesca
- **Co-teachers** – Mari-Berje (board writer) Wilma (bansho whisperer)
- **Knowledge Mobilizer** – Bart
- **Inside Researchers** – ½ the group
- **Outside Researchers** – ½ the group
“Public Research Lesson” Structure

**CO-TEACHING TEAM**
Bansho Whisperer, Board Writer, Lead Voice

**Co-Teaching Facilitator**

**OUTSIDER RESEARCHERS**
Outsider Researcher Facilitator

**INSIDER RESEARCHERS**

- students
- Insider Researcher Facilitator

**INSIDER RESEARCHERS**

- students
- Insider Researcher Facilitator

**INSIDER RESEARCHERS**

- students
- Insider Researcher Facilitator

KKZ, V2, 2012
Lead Voice, Board-Writer, Bansho Whisperer may not tell students what to do or think; they may:

- provide prompts, questions, and circulate together and individually
- gather evidence of student learning through observation (mathematics they see and hear from students)
- redirect students to interact and discuss ideas and questions with one another
- ask students questions to help them clarify and probe their mathematical thinking
- give descriptive feedback to students about their thinking and evidence of learning during After (Consolidation)
Teacher-Researchers (insiders, outsiders)

may NOT talk to students

- **Insiders** - gather and record mathematical evidence of student thinking through observation (what you see, hear), using an assessment for learning seating plan tool for one small group of students

- **Outsiders** - gather and record details of the (a) co-teaching process, evidence co-teaching team uses to make instructional decisions about math content focus and instructional strategies, mathematics teaching details (from students say and teachers say and write)
Record 3 solutions:  
Before (activation), During, (lesson) and After (Practice)  
Record your solutions and add student solutions.  
Sequence and number solutions, 1, 2, 3 …  
Record by number which solution(s) each student did
**Assessment for Learning Seating Plan Tool (p. 2)**

**Record students’ mathematical thinking:**

- what students, say, do (manipulate), and record mathematically.

- student dilemmas

- When students change their mind

- What and when students discover an error or a new idea

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**Key Questions for Analysis of Student Thinking:**

1. Record what you see students doing and saying, mathematically.
2. How are their solutions similar to and different from your solution?
### Assessment for Learning Seating Plan Tool

<table>
<thead>
<tr>
<th>Student Names</th>
<th>During Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jason</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>Morgan</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td>Joe</td>
<td>3, 3, 4</td>
</tr>
</tbody>
</table>

**Mathematical Organizational Criteria**
- Mathematical sequence of solutions
- Learning trajectory

**Mathematical Annotations**
- Concepts - Symbols, labelled diagrams, math strategies - models, calculations, key terms and brief descriptions to elaborate and detail models

**Highlights/Summary (Success Criteria)**
- What do students need to know and be able to do mathematically to achieve the lesson learning goal?
- Learning goal followed by Success criteria and math example

**Lesson Learning Goals:**
- Unpacked from specific curriculum expectations

**Specific Curriculum Expectations from Ontario curriculum**
- Record different solutions to the problem
  - 2 teacher solutions
  - Different student solutions observed during lesson
  - Sequence them mathematically by 1, 2, 3, 4 (numbering labels type of solution) · scaffold towards lesson learning goal
● Students were collaborating – saw them skip counting
● Wrote the multiplication sentence but were stuck explaining after
● Success in math class – proof to explain their thinking
● Maybe too simple?
● Some not listening – discuss with partner – one thing you learned from someone else
● Purpose of working with pairs
● Francesca – felt good
● We could have taped the lesson
Grace Before Meals

Bless us,
O Lord,
and these thy gifts,
which we are about to receive
from thy bounty.
Through Christ our Lord.
Amen.
Reflect on our Collaborative Inquiry Question/s

- What impact will letting students collaborate, make mistakes and try to solve problems by themselves, have on students’ ability to solve multi-step problems?

- How will analyzing students’ solutions and giving them immediate descriptive feedback, impact their ability to independently solve multi-step problems?

- How will building students’ vocabulary impact their ability to communicate their mathematical thinking in French?
What are you going to practise?
Allowing them the opportunity to solve problems without front loading – giving them descriptive feedback
• Students did really well – have to establish some rules re the marker use
• How do we make rich questions?
• Works great
• Wait time so important
• Not enough time to do the practice
• Didn’t try it – can’t break our old habits
What are you going to practise?

Words walls, sentence stems (prompts) – questioning monograph

Je vais soustraire… parce que…

Il va se diriger de deux cases vers la droite, vers le…

Il y a _____ en tout.

J’ai utilisé…
What Classroom Strategies Will We Focus on?

Practice from now until April 4th

What are you going to practise? Same as before

- Allowing students the opportunity to solve problems without front loading – giving them descriptive feedback

- Creating words walls, sentence stems (prompts) – using the questioning monograph
Preparations for Our Next Gr2/3 Study Meeting …

Study Meeting #3 - April 4th, 2014

Read: ppt notes from Session 2

Implement:
• Try two strategies during your math lessons with school colleagues … Practise and reflect.

Bring:
• 4 student solutions to a lesson problem to analyze and discuss during our AM Student Work Sample Analysis
• Your reflections about teaching through problem solving using bansho (board writing)

Dates, Times, Location
#1 – Wed Jan 22nd – Our Lady of Wisdom - Equivalence
#2 – Mon Feb 24th – Our Lady of Wisdom - Multiplication
#3 – Fri April 4th – St Cyril

All sessions are from 8:45 am – 2:45 pm.